Due July 22, 11:59pm on Gradescope.

The following are warm-up exercises and are *not* to be turned in. You may treat these as extra practice problems.

6.3.19, 6.3.21, 6.3.24, 6.4.8, 6.4.25, 6.4.31, 6.5.14, 6.5.29, 6.5.33, 8.5.11, 8.5.15, 8.6.7, 8.6.9.

Turn in the following exercises. Remember to carefully justify every statement that you write, and to follow the style of proper mathematical writing. You may cite any result proved in the textbook or lecture, unless otherwise mentioned. Each problem is worth 10 points with parts weighted equally, unless otherwise mentioned.

1. 6.4.42.

2. **(20 points)** Here is some practice with "algebraically counting in two ways": performing (binomial) expansion in two different ways and then comparing the resulting coefficients.

   (a) **(10 points)** Show that for any positive integer n, $\sum_{k=0}^{2n}(-1)^k\binom{2n}{k}^2 = (-1)^n\binom{2n}{n}$ [Hint: think about $(1-x^2)^{2n}$].

   (b) **(10 points)** Show that if p is a prime and $a > b$ are positive integers, then $\binom{pa}{pb} \equiv \binom{a}{b} \bmod p^2$. [Hint: cleverly expand $(1+x)^{pa} = ((1+x)^p)^a$. In particular, write $(1+x)^p = 1 + x^p + pf(x)$, where $f(x)$ is a polynomial with integer coefficients and $pf(x) = (1+x)^p - 1 - x^p$.]

3. (a) **(5 points)** 6.3.36.

   (b) **(5 points)** 6.5.42.

4. 6.5.48. [Hint: this is tricky. One approach is to think about the *non-chosen* books instead, and think about the possible spacings between those non-chosen books.]

5. 8.5.14.

6. 8.6.10.

7. **(20 points)**

   (a) **(10 points)** 8.6.18.

   (b) **(5 points)** 8.6.19.

   (c) **(5 points)** 8.6.20. More precisely, show that $D_n = n!\sum_{i=0}^{n}(-1)^i/i!$ for $n \geq 0$.

8. **(Bonus Problem, 5 points)** Let n be a positive integer. Show that

$$\sum_{j=0}^{\lfloor n/4 \rfloor} \binom{n}{4j} = 2^{n-2} + 2^{(n-2)/2} \cos\left(\frac{n\pi}{4}\right).$$